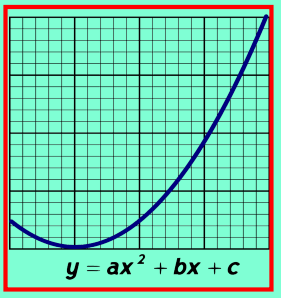
Math 125

Fall 2021

Lecture 51



Class QZ 40

Solve $x^2 + 8x + 25 = 0$ by Completing the Square method.

$x^2 + 8x + 4^2 = -25 + 4^2$ ✓

$(x+4)^2 = -9$ by S.R.M.

$x + 4 = \pm\sqrt{-9}$

$x = -4 \pm \sqrt{9}\sqrt{-1}$

$x = -4 \pm 3i$

✓ $\{-4 \pm 3i\}$

Solve $2x^2 - 7x - 9 = 0$ by Completing the Square method.

Make Lead. Coef. = 1

Divide by 2

$x^2 - \frac{7}{2}x - \frac{9}{2} = 0 \qquad x^2 - \frac{7}{2}x = \frac{9}{2}$

$\frac{1}{2} \cdot \frac{-7}{2} = \frac{-7}{4}$

$x^2 - \frac{7}{2}x + \left(\frac{-7}{4}\right)^2 = \frac{9}{2} + \left(\frac{-7}{4}\right)^2$

$x^2 - \frac{7}{2}x + \frac{49}{16} = \frac{98}{28} + \frac{49}{16}$

$\left(x - \frac{7}{4}\right)^2 = \frac{72}{16} + \frac{49}{16}$

$\left(x - \frac{7}{4}\right)^2 = \frac{121}{16}$

by S.R.M.

$x - \frac{7}{4} = \pm\sqrt{\frac{121}{16}}$

$x = \frac{7}{4} \pm \frac{11}{4}$

$x = \frac{7}{4} + \frac{11}{4} = \frac{18}{4} = \boxed{\frac{9}{2}}$

$x = \frac{7}{4} - \frac{11}{4} = \frac{-4}{4} = \boxed{-1}$

$\Rightarrow \left\{-1, \frac{9}{2}\right\}$

Given $2x^2 - 7x - 9 = 0$

1) Find the value of its discriminant.

$b^2 - 4ac = (-7)^2 - 4(2)(-9) = 49 + 72 = \boxed{121}$

$a = 2,\ b = -7,\ c = -9$

2) Discuss the type of solutions

Since $b^2 - 4ac > 0 \Rightarrow$ Two Real Solutions

3) Use the Quadratic formula to Solve it.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-7) \pm \sqrt{121}}{2(2)} = \frac{7 \pm 11}{4}$

$x = \frac{7+11}{4} = \frac{18}{4} = \boxed{\frac{9}{2}} \qquad x = \frac{7-11}{4} = \frac{-4}{4} = \boxed{-1}$

$\left\{-1, \frac{9}{2}\right\}$

Find a quadratic equation in $ax^2+bx+c=0$ with solutions $-\frac{2}{3}$ and $\frac{1}{5}$.

$x=\frac{-2}{3}$ $\qquad$ $x=\frac{1}{5}$

$3x=-2$ $\qquad$ $5x=1$

$3x+2=0$ $\qquad$ $5x-1=0$

$(3x+2)(5x-1)=0$

$15x^2-3x+10x-2=0$

$15x^2+7x-2=0$

Find a quadratic eqn in $ax^2+bx+c=0$ form with Solutions $-4\pm 3i$.

$x=-4+3i$ $\qquad$ $x=-4-3i$

$x+4-3i=0$ $\qquad$ $x+4+3i=0$

$(x+4-3i)(x+4+3i)=0$

Conjugates

$(x+4)^2-(3i)^2=0$

$(x+4)(x+4)-9i^2=0$

$x^2+4x+4x+16-9(-1)=0$

$x^2+8x+25=0$

Given $f(x) = (x-2)^2 + 3$

$a=1$ $h=2$ $K=3$

1) Discuss directions

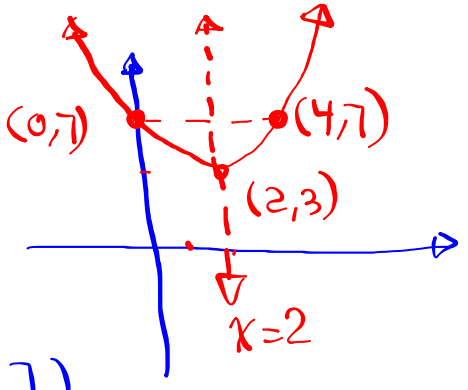
opens upward, a>0

2) Vertex (2,3)

3) A.O.S. $x=2$

4) All intercepts Y-Int (0,7) X-Int None

5) Graph



Domain: $(-\infty, \infty)$

Range: $[3, \infty)$

Given $f(x) = -x^2 + 6x - 9$

$a=-1$ $b=6$ $c=-9$

1) Discuss direction

opens downward, a<0

2) Vertex $h = \frac{-b}{2a} = \frac{-6}{2(-1)} = \frac{-6}{-2} = 3$ $\Rightarrow (3,0)$

3) A.O.S. $K = -(3)^2 + 6(3) - 9 = 0$

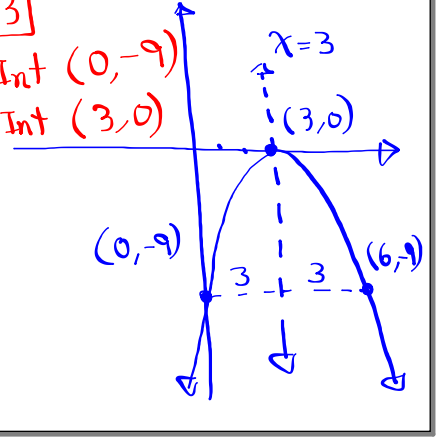
$x = h$ $\boxed{x=3}$

4) All intercepts Y-Int (0,-9) X-Int (3,0)

5) Graph

Domain $(-\infty, \infty)$

Range $(-\infty, 0]$ ✓



Sideway Parabola:

$x = a(y-k)^2 + h$

$a > 0$ ↻ opens right

$a < 0$ ↺ opens left

Vertex (h,k)

A.O.S. $y = k$

x-Int (, 0)

y-Int (0,)

Range $(-\infty, \infty)$

Ex: $x = (y-2)^2 + 3$

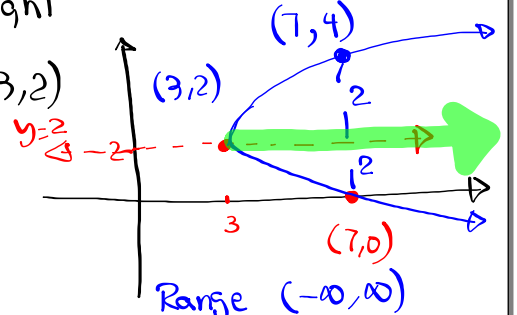
Domain See Graph

$a = 1$ opens right

$h = 3$, $k = 2$ → Vertex $(3,2)$

A.O.S. $y = 2$

x-Int $(7,0)$



Range $(-\infty, \infty)$

Domain $[3, \infty)$

Given $x = -(y+2)^2 + 4$

$x = a(y-k)^2 + h$

$a = -1$

$h = 4$

$k = -2$

opens left

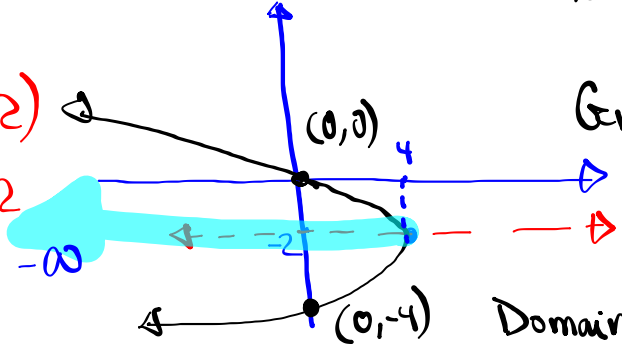
Vertex $(4,-2)$

A.O.S. $y = -2$

y-Int $(0,0)$ $(0,-4)$

x-Int $(0,0)$

Graph



Domain $(-\infty, 4]$

Range $(-\infty, \infty)$

Given $x = -2(y-1)^2 - 4$

$a = -2$ $h = -4$ $k = 1$

Discuss direction: opens left

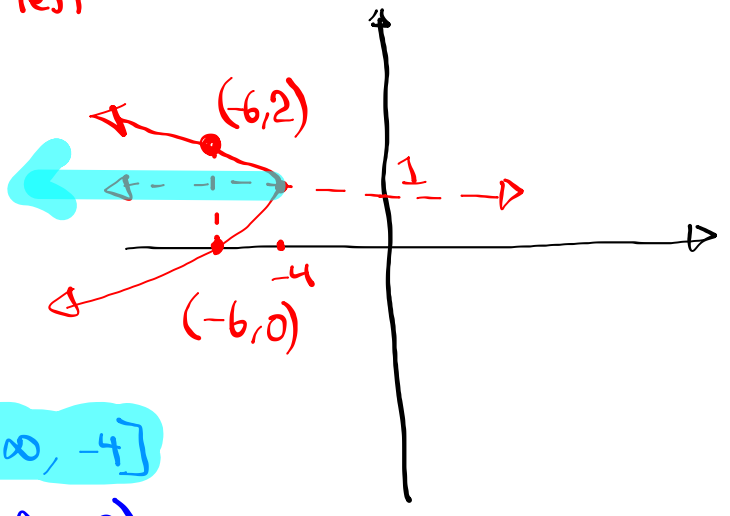
Vertex (-4, 1)

A.O.S. $y = 1$

x-Int (-6, 0)

Y-Int None

Graph



Domain $(-\infty, -4]$

Range $(-\infty, \infty)$

$x = ay^2 + by + c$ type. Tuesday

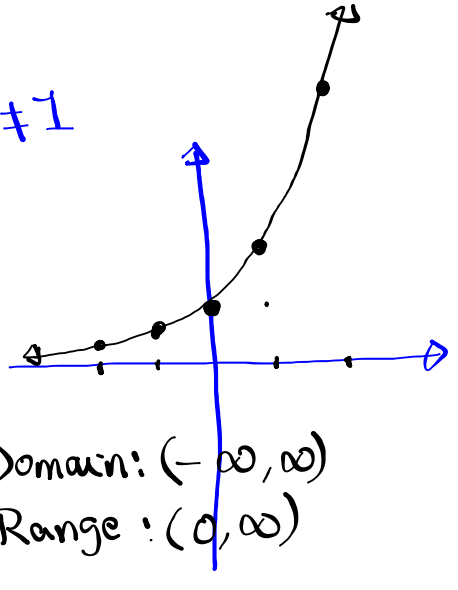
Exponential Function

$f(x) = b^x$, $b > 0$, $b \neq 1$

$f(x) = 2^x$

x	y
0	1
1	2
2	4

x	y
-1	$2^{-1} = \frac{1}{2}$
-2	$2^{-2} = \frac{1}{4}$



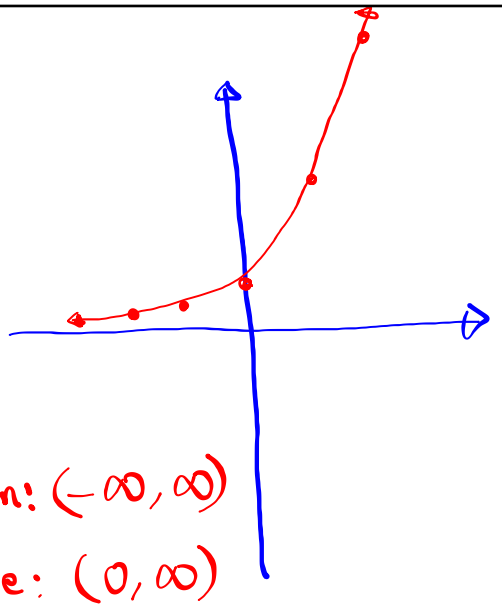
Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

$f(x) = 3^x$

x	y
0	1
1	3
2	9

x	y
-1	$3^{-1} = \frac{1}{3}$
-2	$3^{-2} = \frac{1}{9}$



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Recall

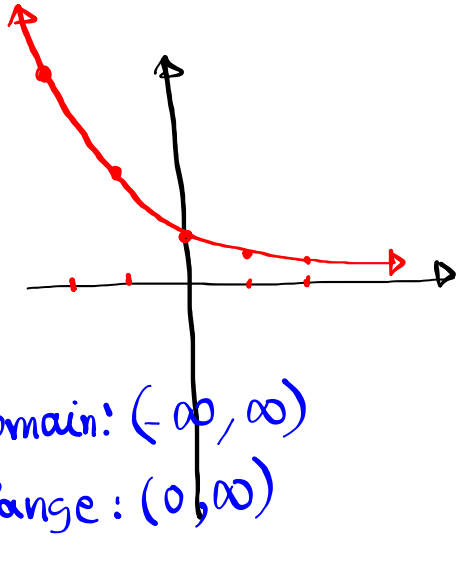
$x^{-n} = \frac{1}{x^n}$

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

$f(x) = \left(\frac{1}{2}\right)^x$

x	y
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

x	y
-1	2
-2	4



Domain: $(-\infty, \infty)$

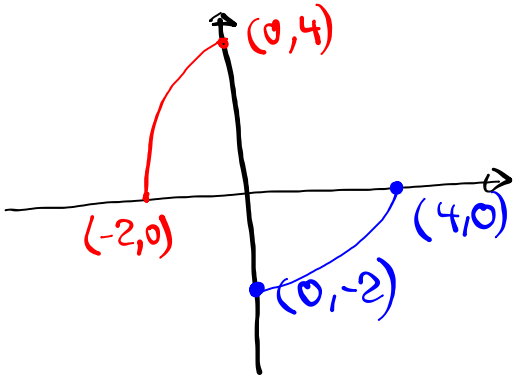
Range: $(0, \infty)$

Recall $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

$\left(\frac{1}{2}\right)^{-1} = \left(\frac{2}{1}\right)^1 = 2$

$\left(\frac{1}{2}\right)^{-2} = \left(\frac{2}{1}\right)^2 = 4$

Consider the graph below



Switch x & y,

draw a new graph

these two graphs

are inverses of

each other.

Class QZ 41

Given $5x^2 - 3x + 10 = 0$

1) Give the value of its discriminant.

$a = 5$

$b = -3$

$c = 10$

$b^2 - 4ac = (-3)^2 - 4(5)(10) = 9 - 200 = \boxed{-191}$

2) Discuss the type of Solutions without Solving.

$b^2 - 4ac < 0 \Rightarrow$ Two imaginary Solutions